Some Results of *k*-Hypergeometric Functions Associated with Integral Transforms and Fractional Calculus

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***Abstract*—In this paper the Laplace Transform, Fractional Fourier Transform, Fractional integration and *k*-Fractional differentiation of the *k*-hypergeometric functions are investigated. An integral representation of some special functions is used to develop these results. Since the results derived here are general in nature, it is expected that they will be a useful addition in the theory of integral transforms and fractional integrals.**

***Index Terms*—*k*-gamma functions, *k*-beta functions, k-hypergeometric functions, *k*-beta transform.**

***MSC 2010 Codes* – 33C05, 44A05**

I. Introduction

MUBEEN and Habibullah [1] defined *k*-hypergeometric functions and k-fractional integrations[2] as a variant of Riemann-Liouville fractional integrals. Diaz and Pariguan [3] have deduced an integral representation of *k*-gamma functions, *k*-beta functions.They have also studied *k*-hypergeometric functions based on Pochhammer *k*-symbols for factorial functions. These studies were extended by Mansour [4], Kokologiannaki [5], Krasniqi [6] and Merovci [7] elaborating and strengthening the scope of *k*-gamma and *k*-beta functions.Romero et.al[8]introduced a new fractional operator called *k*-Riemann-Liouville fractional derivative defined by using the *k*-Gamma functions. They also investigated relationships with the *k*-Riemann Liouville integrals and derived some properties by using Fourier and Laplace transforms. There have been some important generalizations of these functions that have been thoroughly investigated.

II. Definitions

k-Gamma Function

The integral representation of *k*-gamma function[3] is

$$\Gamma_k(x)=\int_0^\infty t^{x-1}e^{-\frac{t^k}{k}}dt, \quad Re(x)>0$$

k-Beta Function

The integral representation of *k*-beta function[3] is

$$B_k(x,y)=\frac{1}{k}\int_0^1 t^{\frac{x}{k}-1}(1-t)^{\frac{y}{k}-1}dt, \quad x>0, y>0$$

The relation between *k*-beta and *k*-gamma function is

$$B_k(x,y)=\frac{\Gamma_k(x)\Gamma_k(y)}{\Gamma_k(x+y)}$$

k- Hypergeometric Function

The *k*-Hypergeometric function defined by Mubeen and Habibullah [1] is

$${}_2F_{1,k}[(\alpha,k),(\beta,k);(\gamma,k);z]=\sum_{n=0}^{\infty}\frac{(\alpha)_{n.k}(\beta)_{n,k}}{(\gamma)_{n,k}}\frac{z^n}{n!} \;, \quad k>0$$

Laplace Transform

The Laplace Transform [9]of *f(t)* is defined as

$$L\{f(t)\}=\int_0^\infty e^{-st}f(t)dt$$

Fractional Fourier Transform

Let f be a function belonging to Lizorkin space ϕ(R) .The Fractional Fourier Transform(FFT) [9] of f of order α is defined as

$$f_\alpha(\omega)=\Im_\alpha[f](\omega)=\int_R e^{i\omega^{\frac{1}{\alpha}}t}f(t)dt$$

k- Fractional Integration

k- Fractional Integration as a modification of Riemann-Liouville fractional integral [2]is defined as

$$I_k^\alpha(f)(x)=\frac{1}{k\Gamma_k(\alpha)}\int_0^x(x-t)^{\frac{\alpha}{k}-1}f(t)dt$$

for $k>0, 0<x<t<\infty$. It reduces to the classical Riemann-Liouville fractional integral by taking limit $k\longrightarrow 1$.

k- Riemann Liouville Fractional Derivative

Let β be a real number, $0<\beta\leq 1$. The *k*-Riemann Liouville Fractional Derivative is given as

$$D_k^\beta f(t)=\frac{\mathrm{d}}{\mathrm{d}t}I_k^{1-\beta}f(t)$$

$$I_k^{1-\beta}f(x)=\frac{1}{k\Gamma_k(1-\beta)}\int_0^x(x-t)^{\frac{1-\beta}{k}-1}f(t)dt$$

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III. Main Results

In this section the new results are found.

Laplace Transform of *k*-Hypergeometric function

If $R(s)>0 \quad and \quad |\frac{y}{ks}|<1$ then

$$L\{z^{\frac{a}{k}-1}{}_2F_{1,k}\left[(\alpha,k),(\beta,k);(\gamma,k);yz\right]\}$$
$$=\frac{k\Gamma_k(a)}{(ks)^{\frac{a}{k}}}{}_3F_{1,k}\left[(\alpha,k),(\beta,k),(a,k)(\gamma,k);\frac{y}{ks}\right]$$

Proof:

$$L\{f(z)\}=\int_0^\infty e^{-sz}f(z)dz$$

$$L\{z^{\frac{a}{k}-1}{}_2F_{1,k}[(\alpha,k),(\beta,k);(\gamma,k);yz]\}$$

$$=\int_0^\infty e^{-sz}z^{\frac{a}{k}-1}{}_2F_{1,k}[(\alpha,k),(\beta,k);(\gamma,k);yz]dz$$

$$=\int_0^\infty e^{-sz}z^{\frac{a}{k}-1}\sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}(\beta)_{n,k}}{(\gamma)_{n,k}}\frac{(yz)^n}{n!}dz$$

$$=\sum_{n=0}^{\infty}\left[\frac{\Gamma_k(\alpha+nk)\Gamma_k(\beta+nk)\Gamma_k(\gamma)}{\Gamma_k(\alpha)\Gamma_k(\beta)\Gamma_k(\gamma+nk)}\frac{y^n}{n!}\right]$$
$$\times\left[\int_0^\infty e^{-sz}z^{\frac{a}{k}+n-1}dz\right]$$

$$=\sum_{n=0}^{\infty}\left[\frac{\Gamma_k(\alpha+nk)\Gamma_k(\beta+nk)\Gamma_k(\gamma)}{\Gamma_k(\alpha)\Gamma_k(\beta)\Gamma_k(\gamma+nk)}\frac{y^n}{n!}\right]$$
$$\times\left[\int_0^\infty e^{-\frac{t^k}{k}}\left(\frac{t^k}{ks}\right)^{\frac{a}{k}+n-1}\frac{t^{k-1}}{s}dt\right]$$

$$\left[Putting \quad sz=\frac{t^k}{k}\right]$$

$$=\sum_{n=0}^{\infty}\left[\frac{\Gamma_k(\alpha+nk)\Gamma_k(\beta+nk)\Gamma_k(\gamma)}{\Gamma_k(\alpha)\Gamma_k(\beta)\Gamma_k(\gamma+nk)}\frac{y^n}{n!}dt\right]$$
$$\times\left[\int_0^\infty\frac{e^{-\frac{t^k}{k}}t^{a+nk-1}}{k^{\frac{a}{k}+n-1}s^{\frac{a}{k}+n}}dt\right]$$

$$=\sum_{n=0}^{\infty}\left[\frac{\Gamma_k(\alpha+nk)\Gamma_k(\beta+nk)\Gamma_k(\gamma)\Gamma_k(a+nk)}{\Gamma_k(\alpha)\Gamma_k(\beta)\Gamma_k(\gamma+nk)k^{\frac{a}{k}-1}s^{\frac{a}{k}}}\right.$$
$$\left.\cdot\left(\frac{y}{ks}\right)^n\frac{1}{n!}\right]$$

$$=\frac{k\Gamma_k(a)}{(ks)^{\frac{a}{k}}}\sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}(\beta)_{n,k}(a)_{n,k}}{(\gamma)_{n,k}}\left(\frac{y}{ks}\right)^n\frac{1}{n!}$$

$$=\frac{k\Gamma_k(a)}{(ks)^{\frac{a}{k}}}{}_3F_{1,k}\left[(\alpha,k),(\beta,k),(a,k)(\gamma,k);\frac{y}{ks}\right]$$

Special case: For k=1, the result shows Laplace Transform of Hypergeometric function

$$L\{z^{\alpha-1}{}_2F_1\left[\alpha,\beta;\gamma;yz\right]\}=\frac{\Gamma(a)}{s^a}{}_3F_1\left[\alpha,\beta,a;\gamma;\frac{y}{s}\right]$$

Fractional Integration of *k*-Hypergeometric function

Let $\rho,\mu,\nu,\eta,\delta\in\mathbb{C}$ and $k\in\mathbb{R}^+$ be such that Re$(\gamma)>0$, Re$\left(\frac{\lambda}{k}\right)>$ max. $\{0,\text{Re}(\mu-\eta),\text{Re}(\rho+\mu+\nu-\delta)\}$, then for $x>0$

$$\left[I_{0+}^{\rho,\mu,\nu,\eta,\delta}\left(z^{\frac{\lambda}{k}-1}{}_2F_{1,k}\left[(\alpha,k),(\beta,k);(\gamma,k);z\right]\right)\right](x)$$

$$=k^{\delta}x^{-\rho-\mu+\delta+\frac{\lambda}{k}-1}\frac{\Gamma_k(\lambda)\Gamma_k(\lambda-\mu k+\eta k)}{\Gamma_k(\lambda+\eta k)\Gamma(\lambda-\rho k-\mu k+\delta k)}$$
$$\times\frac{\Gamma_k(\lambda-\rho k-\mu k-\nu k+\delta k}{\Gamma_k(\lambda-\mu k-\nu k+\delta k)}$$

$$\times{}_5F_4\left[\begin{matrix}(\alpha,k),(\beta,k),(\lambda,k)(\lambda-\mu k+\eta k,k),\\(\lambda-\rho k-\mu k-\nu k+\delta k,k);\\ & x\\(\gamma,k),(\lambda+\eta k,k),(\lambda-\rho k-\mu k+\delta k,k),\\(\lambda-\mu k-\nu k+\delta k,k);\end{matrix}\right]$$

Proof: By Saigo and Maeda [10],

$$I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}x^{\rho-1}=\frac{\Gamma(\rho)\Gamma(\rho+\gamma-\alpha-\alpha'-\beta)}{\Gamma(\rho+\gamma-\alpha-\alpha')\Gamma(\rho+\beta')}$$
$$\times\frac{\Gamma(\rho+\beta'-\alpha')}{\Gamma(\rho+\gamma-\alpha'-\beta)}x^{\rho-\alpha-\alpha'+\gamma-1}$$

$$\left[I_{0+}^{\rho,\mu,\nu,\eta,\delta}\left(z^{\frac{\lambda}{k}-1}{}_2F_{1,k}\left[(\alpha,k),(\beta,k);(\gamma,k);z\right]\right)\right](x)$$

$$=\left[I_{0+}^{\rho,\mu,\nu,\eta,\delta}\left\{z^{\frac{\lambda}{k}-1}\sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}(\beta)_{n,k}}{(\gamma)_{n,k}}\left(\frac{z^n}{n!}\right)\right\}\right](x)$$

$$=\sum_{n=0}^{\infty}\frac{\Gamma_k(\alpha+nk)\Gamma_k(\beta+nk)\Gamma_k(\gamma)}{\Gamma_k(\alpha)\Gamma_k(\beta)\Gamma_k(\gamma+nk)}\frac{1}{n!}\left(I_{0+}^{\rho,\mu,\nu,\eta,\delta}z^{\frac{\lambda}{k}+n-1}\right)(x)$$

$$=\sum_{n=0}^{\infty}\frac{\Gamma_k(\alpha+nk)\Gamma_k(\beta+nk)\Gamma_k(\gamma)}{\Gamma_k(\alpha)\Gamma_k(\beta)\Gamma_k(\gamma+nk)}\frac{1}{n!}\frac{\Gamma\left(\frac{\lambda}{k}+n\right)}{\Gamma\left(\eta+\frac{\lambda}{k}+n\right)}$$
$$\times\frac{\Gamma\left(-\mu+\eta+\frac{\lambda+nk}{k}\right)}{\Gamma\left(-\rho-\mu+\delta+\frac{\lambda+nk}{k}\right)}$$
$$\times\frac{\Gamma\left(-\rho-\mu-\nu+\delta+\frac{\lambda+nk}{k}\right)}{\Gamma\left(-\mu-\nu+\delta+\frac{\lambda+nk}{k}\right)}x^{-\rho-\mu+\delta+\frac{\lambda+nk}{k}-1}$$

$$=\sum_{n=0}^{\infty}\frac{\Gamma_k(\alpha+nk)\Gamma_k(\beta+nk)\Gamma_k(\gamma)}{\Gamma_k(\alpha)\Gamma_k(\beta)\Gamma_k(\gamma+nk)}\frac{1}{n!}\frac{\Gamma\left(\frac{\lambda+nk}{k}\right)}{\Gamma\left(\frac{\eta k+\lambda+nk}{k}\right)}$$

$$\times\frac{\Gamma\left(\frac{-\mu k+\eta k+\lambda+nk}{k}\right)}{\Gamma\left(\frac{-\rho k-\mu k+\delta k+\lambda+nk}{k}\right)}$$

$$\times\frac{\Gamma\left(\frac{-\rho k-\mu k-\nu k+\delta k+\lambda+nk}{k}\right)}{\Gamma\left(\frac{-\mu k-\nu k+\delta k+\lambda+nk}{k}\right)}x^{-\rho-\mu+\delta+\frac{\lambda}{k}+n-1}$$

$$=\sum_{n=0}^{\infty}\frac{\Gamma_k(\alpha+nk)\Gamma_k(\beta+nk)\Gamma_k(\gamma)}{\Gamma_k(\alpha)\Gamma_k(\beta)\Gamma_k(\gamma+nk)}\frac{x^n}{n!}$$

$$\times\frac{\Gamma_k(\lambda+nk)k^{\frac{\eta k+\lambda+nk}{k}-1}}{k^{\frac{\lambda+nk}{k}}\Gamma_k(\eta k+\lambda+nk)}$$

$$\times\frac{\Gamma_k(-\mu k+\eta k+\lambda+nk)}{k^{\frac{-\mu k+\eta k+\lambda+nk}{k}-1}}$$

$$\times\frac{\Gamma_k(-\rho k-\mu k-\nu k+\delta k+\lambda+nk)}{k^{\frac{-\rho k-\mu k-\nu k+\delta k+\lambda+nk}{k}-1}}$$

$$\times\frac{k^{\frac{-\rho k-\mu k+\delta k+\lambda+nk}{k}-1}}{\Gamma_k(-\rho k-\mu k+\delta k+\lambda+nk)}$$

$$\times\frac{k^{\frac{-\mu k-\nu k+\delta k+\lambda+nk}{k}-1}}{\Gamma_k(-\mu k-\nu k+\delta k+\lambda+nk)}x^{-\rho-\mu+\delta+\frac{\lambda}{k}-1}$$

$$=\sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}(\beta)_{n,k}}{(\gamma)_{n,k}}\frac{x^n}{n!}k^{\delta}\frac{(\lambda)_{n,k}\Gamma_k(\lambda)}{(\lambda+\eta k)_{n,k}\Gamma_k(\lambda+\eta k)}$$

$$\times\frac{(\lambda-\mu k+\eta k)_{n,k}\Gamma_k(-\mu k+\eta k+\lambda)}{(\lambda-\rho k-\mu k+\delta k)_{n,k}\Gamma_k(\lambda-\rho k-\mu k+\delta k)}$$

$$\times\frac{(\lambda-\rho k-\mu k+\delta k)_{n,k}}{(\lambda-\mu k-\nu k+\delta k)_{n,k}}$$

$$\times\frac{\Gamma_k(\lambda-\rho k-\mu k-\nu k+\delta k)}{\Gamma_k(\lambda-\mu k-\nu k+\delta k)}x^{-\rho-\mu+\delta+\frac{\lambda}{k}-1}$$

$$=k^{\delta}x^{-\rho-\mu+\delta+\frac{\lambda}{k}-1}\frac{\Gamma_k(\lambda)\Gamma_k(\lambda-\mu k+\eta k)}{\Gamma_k(\lambda+\eta k)\Gamma_k(\lambda-\rho k-\mu k+\delta k)}$$

$$\times\frac{\Gamma_k(\lambda-\rho k-\mu k-\nu k+\delta k)}{\Gamma_k(\lambda-\mu k-\nu k+\delta k)}$$

$$\times{}_5F_4\left[\begin{array}{r}(\alpha,k),(\beta,k),(\lambda,k)(\lambda-\mu k+\eta k,k),\\(\lambda-\rho k-\mu k-\nu k+\delta k,k);\\x\\(\gamma,k),(\lambda+\eta k,k),(\lambda-\rho k-\mu k+\delta k,k),\\(\lambda-\mu k-\nu k+\delta k,k);\end{array}\right]$$

Special case: For *k*=1, the result shows Fractional integration of Hypergeometric function

$$\left[I_{0+}^{\rho,\mu,\nu,\eta,\delta}\left(z^{\lambda-1}{}_2F_1[\alpha,\beta;\gamma;z]\right)\right](x)$$

$$=x^{-\rho-\mu+\delta+\lambda-1}\frac{\Gamma(\lambda)\Gamma(\lambda-\mu+\eta)}{\Gamma(\lambda+\eta)\Gamma(\lambda-\rho-\mu+\delta)}$$

$$\times\frac{\Gamma(\lambda-\rho-\mu-\nu+\delta)}{\Gamma(\lambda-\mu-\nu+\delta)}$$

$$\times{}_5F_4\left[\begin{array}{r}\alpha,\beta,\lambda,\lambda-\mu+\eta,\lambda-\rho-\mu-\nu+\delta;\\x\\\gamma,\lambda+\eta,\lambda-\rho-\mu+\delta,\lambda-\mu-\nu+\delta;\end{array}\right]$$

Fractional Fourier Transform of *k*-Hypergeometric function

Fractional Fourier Transform of *k*-Hypergeometric functionof order α is

$$\Im_{\alpha}\left[{}_2F_{1,k}\{(\alpha,k),(\beta,k);(\gamma,k);z\}\right]$$

$$=\sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}(\beta)_{n,k}(-1)^n(i)^{-(n+1)}(\omega)^{-(\frac{n+1}{\alpha})}}{(\gamma)_{n,k}n!}\Gamma(n+1)$$

Proof: The Fractional Fourier Transform of *k*-hypergeometric function for $z<0$ is

$$\Im_{\alpha}\left[{}_2F_{1,k}\{(\alpha,k),(\beta,k);(\gamma,k);z\}\right]$$

$$=\int_R e^{i\omega^{\frac{1}{\alpha}}z}\sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}(\beta)_{n,k}}{(\gamma)_{n,k}}\frac{z^n}{n!}dz$$

$$=\sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}(\beta)_{n,k}}{(\gamma)_{n,k}n!}\int_{-\infty}^{0}e^{i\omega^{\frac{1}{\alpha}}z}z^n dz$$

$$=\sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}(\beta)_{n,k}}{(\gamma)_{n,k}n!}\int_{0}^{\infty}e^{-t}\left(-\frac{t}{i\omega^{\frac{1}{\alpha}}}\right)^n\left(\frac{dt}{i\omega^{\frac{1}{\alpha}}}\right)$$

$$=\sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}(\beta)_{n,k}(i)^{n-1}(\omega)^{-(\frac{n+1}{\alpha})}}{(\gamma)_{n,k}n!}$$

Special case: For *k*=1, the result shows Fractional Fourier Transform of Hypergeometric function

$$\Im_{\alpha}\left[{}_2F_1\{\alpha,\beta;\gamma;z\}\right]$$

$$=\sum_{n=0}^{\infty}\frac{(\alpha)_n(\beta)_n(-1)^n(i)^{-(n+1)}(\omega)^{-(\frac{n+1}{\alpha})}}{(\gamma)_n n!}\Gamma(n+1)$$

***k*-Fractional Differentiation of *k*-Hypergeometric function**

If μ be a real number and $0<\mu\leq 1$,

$$D_k^{\mu}\left[z^{\frac{\lambda}{k}}{}_2F_{1,k}\{(\alpha,k),(\beta,k);(\gamma,k);z\}\right]=\frac{\lambda\Gamma_k(\lambda)z^{\frac{1-\mu+\lambda}{k}-1}}{k\Gamma_k(1-\mu+\lambda)}$$

$$\times {}_3F_{2,k}\left[\begin{matrix}(\alpha,k),(\beta,k),(\lambda+k,k); \\ (\gamma,k),(1-\mu+\lambda,k);\end{matrix}\; z\right]$$

Proof:

$$D_k^{\mu}\left[z^{\frac{\lambda}{k}}{}_2F_{1,k}\{(\alpha,k),(\beta,k);(\gamma,k);z\}\right]$$

$$=\frac{d}{dz}\left[I_k^{1-\mu}z^{\frac{\lambda}{k}}{}_2F_{1,k}\{(\alpha,k),(\beta,k);(\gamma,k);z\}\right]$$

$$=\frac{d}{dz}\frac{1}{k\Gamma_k(1-\mu)}$$

$$\times\int_0^z (z-t)^{\frac{1-\mu}{k}-1}t^{\frac{\lambda}{k}}{}_2F_{1,k}[(\alpha,k),(\beta,k);(\gamma,k);t]dt$$

$$=\frac{1}{k\Gamma_k(1-\mu)}\frac{d}{dz}\int_0^z (z-t)^{\frac{1-\mu}{k}-1}t^{\frac{\lambda}{k}}\sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}(\beta)_{n,k}}{(\gamma)_{n,k}}\frac{t^n}{n!}dt$$

$$=\frac{1}{k\Gamma_k(1-\mu)}\sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}(\beta)_{n,k}}{n!(\gamma)_{n,k}}\frac{d}{dz}\int_0^1 (z-zx)^{\frac{1-\mu}{k}-1}(zx)^{n+\frac{\lambda}{k}}z\,dx$$

[Putting $t=zx$]

$$=\frac{1}{k\Gamma_k(1-\mu)}\sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}(\beta)_{n,k}}{n!(\gamma)_{n,k}}$$

$$\times\frac{d}{dz}\int_0^1 (1-x)^{\frac{1-\mu}{k}-1}(x)^{n+\frac{\lambda}{k}}z^{\frac{1-\mu+\lambda+nk}{k}}\;dx$$

$$=\frac{1}{k\Gamma_k(1-\mu)}\sum_{n=0}^{\infty}\left[\frac{(\alpha)_{n,k}(\beta)_{n,k}}{n!(\gamma)_{n,k}}\right.$$

$$\left.\times\frac{d}{dz}z^{\frac{1-\mu+\lambda+nk}{k}}B\Big(\frac{1-\mu}{k},n+\frac{\lambda}{k}+1\Big)\right]$$

$$=\frac{1}{k\Gamma_k(1-\mu)}\sum_{n=0}^{\infty}\left[\frac{(\alpha)_{n,k}(\beta)_{n,k}}{n!(\gamma)_{n,k}}\left(\frac{1-\mu+\lambda+nk}{k}\right)\right.$$

$$\left.\times z^{\frac{1-\mu+\lambda+nk}{k}-1}\frac{\Gamma\left(\frac{1-\mu}{k}\right)\Gamma\left(\frac{nk+\lambda+k}{k}\right)}{\left(\frac{1-\mu+\lambda+nk}{k}+1\right)}\right]$$

$$=\frac{1}{k\Gamma_k(1-\mu)}\sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}(\beta)_{n,k}}{n!(\gamma)_{n,k}}\left(\frac{1-\mu+\lambda+nk}{k}\right)$$

$$\times z^{\frac{1-\mu+\lambda+nk}{k}-1}\frac{\Gamma\left(\frac{1-\mu}{k}\right)\Gamma\left(\frac{nk+\lambda+k}{k}\right)}{\left(\frac{1-\mu+\lambda+nk}{k}\right)\Gamma\left(\frac{1-\mu+\lambda+nk}{k}\right)}$$

$$=\frac{1}{k\Gamma_k(1-\mu)}\sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}(\beta)_{n,k}}{n!(\gamma)_{n,k}}z^{\frac{1-\mu+\lambda+nk}{k}-1}$$

$$\times\frac{\Gamma_k(1-\mu)\Gamma_k(nk+\lambda+k)k^{\frac{1-\mu+\lambda+nk}{k}-1}}{k^{\frac{1-\mu}{k}-1}k^{\frac{nk+\lambda+k}{k}}\Gamma_k(1-\mu+\lambda+nk)}$$

$$=\sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}(\beta)_{n,k}}{n!(\gamma)_{n,k}}z^{\frac{1-\mu+\lambda+nk}{k}-1}\frac{\Gamma_k(nk+\lambda+k)}{k\Gamma_k(1-\mu+\lambda+nk)}$$

$$=\sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}(\beta)_{n,k}}{n!(\gamma)_{n,k}}\frac{(\lambda+k)_{n,k}}{k(1-\mu+\lambda)_{n,k}}$$

$$\times\frac{\Gamma_k(\lambda+k)}{\Gamma_k(1-\mu+\lambda)}z^{\frac{1-\mu+\lambda}{k}-1}z^n$$

$$=z^{\frac{1-\mu+\lambda}{k}-1}\frac{\lambda\Gamma_k(\lambda)}{k\Gamma_k(1-\mu+\lambda)}\sum_{n=0}^{\infty}\frac{(\alpha)_{n,k}(\beta)_{n,k}(\lambda+k)_{n,k}}{n!(\gamma)_{n,k}(1-\mu+\lambda)_{n,k}}z^n$$

$$=z^{\frac{1-\mu+\lambda}{k}-1}\frac{\lambda\Gamma_k(\lambda)}{k\Gamma_k(1-\mu+\lambda)}\times{}_3F_{2,k}\left[\begin{matrix}(\alpha,k),(\beta,k),(\lambda+k,k); \\ (\gamma,k),(1-\mu+\lambda,k);\end{matrix}\; z\right]$$

Special case: For k=1, the result shows Fractional Differentiation of Hypergeometric function

$$D^{\mu}[z^{\lambda}{}_2F_1\{\alpha,\beta;\gamma;z\}]$$

$$=\frac{\lambda\Gamma(\lambda)z^{-\mu+\lambda}}{\Gamma(1-\mu+\lambda)}{}_3F_2\left[\begin{matrix}\alpha,\beta,(\lambda+1); \\ \gamma,(1-\mu+\lambda);\end{matrix}\; z\right]$$

IV. Conclusion

In this paper, some useful results and relations are derived using integral representation of generalized k-hypergeometric functions. An attempt is made to derive k-Beta transform of k-hypergeometric functions. These results can be further extended by special choices of the parameters.

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